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A QUANTITATIVE CRITERION FOR DEFINING PLANETS

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ABSTRACT

A simple metric can be used to determine whether a planet or exoplanet can clear its orbital zone during a characteristic time scale, such as the lifetime of the host star on the main sequence. This criterion requires only estimates of star mass, planet mass, and orbital period, making it possible to immediately classify 99% of all known exoplanets. All eight planets and all classifiable exoplanets satisfy the criterion. This metric may be useful in generalizing and simplifying the definition of a planet.

Key words: celestial mechanics – gravitation – planets and satellites: dynamical evolution and stability – planets and satellites: fundamental parameters – planets and satellites: general – standards

1. INTRODUCTION

In 2006, the International Astronomical Union (IAU) adopted resolution B5, which states: "A planet is a celestial body that (a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, and (c) has cleared the neighbourhood around its orbit."

Here, we propose a simple metric that allows for the quantification of the third requirement and the extension of the definition to planets orbiting other stars.

It must be emphasized at the outset that a planet can never completely clear its orbital zone, because gravitational and radiative forces continually perturb the orbits of asteroids and comets into planet-crossing orbits. What the IAU intended is not the impossible standard of impeccable orbit clearing; rather the standard is analogous to what Soter (2006, 2008) described as a *dynamical-dominance* criterion. In this article, we use the IAU orbit-clearing language even though the dynamical-dominance language seems less prone to misinterpretation.

2. EXISTING METRIC

We seek to determine whether a celestial body can clear the neighborhood around its orbit. To do so, we adapt the criterion that Tremaine (1993) developed for the formation of Oort-type comet clouds. He considered the ejection of comets by a single planet of mass M_p on a circular orbit of radius a_p around a central star of mass $M_\star$. The ejection process is a diffusion or random walk process in the comet's orbital energy, with the energy described by the variable $x = 1/a$ where a is the semimajor axis of the comet's orbit. The diffusion coefficient $D_x = \langle(\Delta x)^2\rangle^{1/2}$ is the root mean square change in x per orbit resulting from gravity kicks from the planet. Based on earlier work by Fernandez (1981) and Duncan et al. (1987), Tremaine (1993) found

$$D_x \simeq \frac{10}{a_\mathrm{p}}\frac{M_\mathrm{p}}{M_\star}. \tag{1}$$

The characteristic number of orbits required for the energy to change by an amount equal to itself is (x^2/D_x^2) and the corresponding diffusion time is $t_\mathrm{diff} = P(x^2/D_x^2)$, where P is the comet's orbital period. Tremaine (1993) considered comets on orbits initially similar to the orbit of the planet, i.e., $x = a_\mathrm{p}^{-1}$, and computed the planet mass required for the comet diffusion time to be less than the age of the planetary system $t_\star$. He found

$$\frac{M_\mathrm{p}}{M_\oplus} \gtrsim \left(\frac{M_\star}{M_\odot}\right)^{3/4}\left(\frac{t_\star}{10^9\ \mathrm{years}}\right)^{-1/2}\left(\frac{a_\mathrm{p}}{1\ \mathrm{au}}\right)^{3/4}, \tag{2}$$

where the symbols $\oplus$ and $\odot$ refer to Earth and Sun, respectively. This criterion has the same dependence as the criterion $\Lambda \propto M_\mathrm{p}^2/P$ that has been considered by others (Stern & Levison 2002; Soter 2006).

3. PROPOSED METRIC

By requiring a change in energy equal to the initial orbital energy, Tremaine (1993) constructed a condition that ensures that small bodies are scattered out to very large distances, which is the proper criterion when contemplating the formation of Oort-type comet clouds. Here, we are interested in a criterion that meets the third requirement of the 2006 IAU planet definition, i.e., that a planet clears its orbital zone. Ejection is not required. Instead, what is needed is a change in orbital energy that is sufficient to evacuate the small bodies out to a certain distance CR_H, where C is a numerical constant and R_H is the Hill radius of the planet:

$$R_\mathrm{H} = \left(\frac{M_\mathrm{p}}{3M_\star}\right)^{1/3} a_\mathrm{p}. \tag{3}$$

The value of C must exceed $2\sqrt{3}$ to ensure that the planet clears its feeding zone (Birn 1973; Artymowicz 1987; Gladman 1993; Ida & Makino 1993). A more stringent condition would impose clearing a zone extending to 5 Hill radii ($C = 5$). The latter value mirrors certain stability criteria and the observed dynamical spacing between exoplanets (Section 7).

Consider small bodies initially on orbits similar to the orbit of the planet, with $a \simeq a_\mathrm{p}$. The required energy change for clearing a region of size CR_H around the orbit is

$$\delta x = \frac{1}{a_\mathrm{p}} - \frac{1}{a_\mathrm{p} + CR_\mathrm{H}} = \frac{CR_\mathrm{H}/a_\mathrm{p}}{a_\mathrm{p}\left(1 + CR_\mathrm{H}/a_\mathrm{p}\right)}. \tag{4}$$

Ignoring the second term in the denominator provides a lower bound on the energy requirement and an excellent approximation in most situations:

$$\delta x \simeq \frac{C}{a_p}\left(\frac{M_p}{3M_\star}\right)^{1/3}. \quad (5)$$

We can define the clearing time as

$$t_{\rm clear} = P\frac{\delta x^2}{D_x^2}, \quad (6)$$

and use Equations (1) and (5) with the orbital period $P = 2\pi a_p^{3/2}/(GM_\star)^{1/2}$ to arrive at

$$t_{\rm clear} = C^2 1.1 \times 10^5 \text{ years}\left(\frac{M_\star}{M_\odot}\right)^{5/6}\left(\frac{M_p}{M_\oplus}\right)^{-4/3}\left(\frac{a_p}{1\text{ au}}\right)^{3/2}. \quad (7)$$

An Earth-mass planet orbiting a solar-mass star at 1 au can clear its orbital zone to $2\sqrt{3}$ Hill radii in $\sim$1 Myr.

In the spirit of the IAU resolution, we suggest that a body that is capable of clearing its orbit within a well-defined time interval is a planet. Requiring that the clearing time be less than $t_\star$, which is now understood as a characteristic time related to the host star, we find an expression for the minimum orbit-clearing mass:

$$\frac{M_p}{M_\oplus} \gtrsim C^{3/2}\left(\frac{M_\star}{M_\odot}\right)^{5/8}\left(\frac{t_\star}{1.1 \times 10^5\text{ years}}\right)^{-3/4}\left(\frac{a_p}{1\text{ au}}\right)^{9/8}. \quad (8)$$

This relationship clearly distinguishes the eight planets in the solar system from all other bodies (Figures 1 and 2).

For main sequence stars, a sensible characteristic time scale is the host star's lifetime on the main sequence, i.e., $t_\star \simeq t_{\rm MS}$. Incorporating the approximate relationship $t_{\rm MS}/t_\odot \propto (M_\star/M_\odot)^{-2.5}$ with $t_\odot = 10^{10}$ years into Equation (8), we find

$$\frac{M_p}{M_\oplus} \gtrsim 1.9 \times 10^{-4} C^{3/2}\left(\frac{M_\star}{M_\odot}\right)^{5/2}\left(\frac{a_p}{1\text{ au}}\right)^{9/8}. \quad (9)$$

For most stars of interest, the main sequence lifetime has uncertainties up to a factor of 2 and the corresponding uncertainty on the orbit-clearing mass is <2.

We use the notation $M_{\rm clear}$ to represent the orbit-clearing mass given by the right-hand side of Equation (8) or (9) and we use the symbol Π to represent the mass of a planetary body in terms of the corresponding orbit-clearing mass:

$$\Pi = \frac{M_{\rm body}}{M_{\rm clear}}. \quad (10)$$

A simple planet test consists of evaluating whether the discriminant Π exceeds 1. Values of Π for solar system bodies are listed in Table 1 and shown in Figure 2.

The proposed metric for classifying planets is attractive because it relies solely on properties that are typically known (i.e., host star mass) or observable from Earth shortly after discovery (i.e., planet mass and semimajor axis or orbital period). When the planet mass is not directly available, other observables (e.g., planet radius) can be used to place bounds on M_p. In the next section, we use Equation (8) or (9) to test whether known exoplanets can clear their orbits.

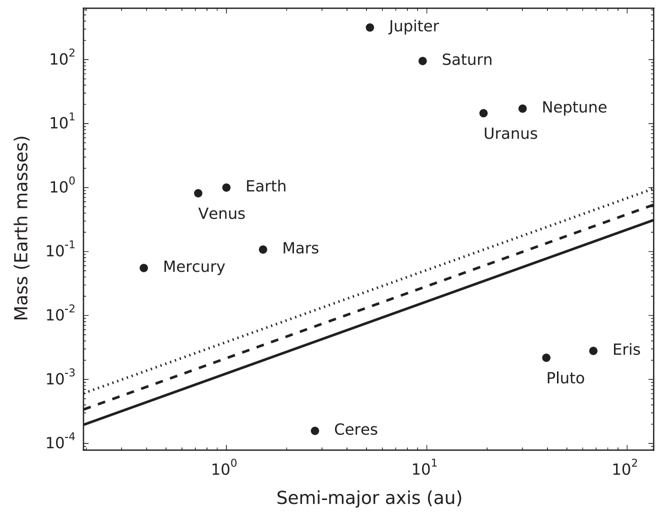

Figure 1. Mass required to clear an orbital zone as a function of semimajor axis for a host star of mass 1 $M_\odot$. The top two lines show clearing to 5 Hill radii in either 10 billion years (dashed line) or 4.6 billion years (dotted line). The solid line shows clearing of the feeding zone ($2\sqrt{3}$ Hill radii) in 10 billion years.

Table 1
Values of the Planet Discriminant Π ($C = 2\sqrt{3}$, $t_\star = t_{\rm MS}$) for Solar System Bodies

Body	Mass ($M_\oplus$)	Π
Jupiter	317.90	4.0×10^4
Saturn	95.19	6.1×10^3
Venus	0.815	9.5×10^2
Earth	1.000	8.1×10^2
Uranus	14.54	4.2×10^2
Neptune	17.15	3.0×10^2
Mercury	0.055	1.3×10^2
Mars	0.107	5.4×10^1
Ceres	1.6×10^{-4}	4.0×10^{-2}
Pluto	2.2×10^{-3}	2.8×10^{-2}
Eris	2.8×10^{-3}	2.0×10^{-2}

4. CLASSIFICATION OF EXOPLANETS

We applied the proposed planet criterion to the exoplanets listed in the NASA Exoplanet Archive.[1] We were able to classify 4620/4664 (99%) of Kepler objects, 829/849 (98%) of non-Kepler objects, and 5/5 (100%) of pulsar objects.

4.1. Kepler Objects

As of 2015 July 17, the archive contained 4664 Kepler Objects of Interest (KOI) that were not labeled as false positive. Of those, 1001 were marked as confirmed and 3663 were marked as candidates. The archive provided mass estimates for the host stars of 4135 KOIs. For the remaining objects, we computed stellar mass estimates on the basis of log g and stellar radius, when available. This process yielded a total of 4620 classifiable Kepler objects, after excluding one object (K07571.01) that was listed with a planet radius equal to 0.

When the planet mass was not available in the archive, we applied radius–mass relationships within their domain of applicability to convert the Kepler measurements of planet radius to estimates of planet mass: Fang & Margot (2012,

[1] http://exoplanetarchive.ipac.caltech.edu

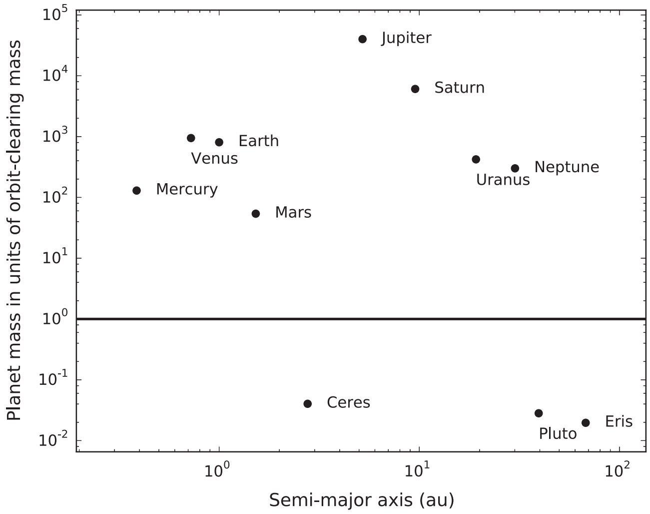

Figure 2. Planet test ($C = 2\sqrt{3}$, $t_\star = t_{MS}$) applied to objects in the solar system. All eight planets have a mass that exceeds the mass required to clear the corresponding orbital zone ($\Pi \geqslant 1$).

$R_p < 25R_\oplus$), Wu & Lithwick (2013, $R_p < 11R_\oplus$), Weiss & Marcy (2014, $R_p < 4R_\oplus$), Fabrycky et al. (2014), Wolfgang et al. (2015, $R_p < 4R_\oplus$). We used the resulting values in Equation (9) to test whether each KOI has sufficient mass to clear its orbit. We found that all KOIs satisfy the criterion irrespective of the choice of the radius–mass relationship (Figure 3). For ease of presentation, we eliminated from the figure one object (K01174.01) listed with an orbital period of 356 years; its mass exceeds the orbit-clearing mass by a factor of ~200. We also eliminated 343 objects listed with a planet radius exceeding $20R_\oplus$, leaving a total of 4276 KOIs for display. A trend of slope ~−1 is visible in the figure because the lower limit on KOI masses does not vary substantially with a whereas the orbit-clearing mass scales as $a^{9/8}$.

4.2. Non-Kepler Exoplanets

As of 2015 July 17, the archive contained 1877 confirmed exoplanets. We treated KOIs and five pulsar planets separately (Sections 4.1 and 4.3, respectively). Eliminating names that include "Kepler" or "KOI" and the pulsar planets yielded a sample of 849 objects. The archive provided mass estimates for the host stars of 696 exoplanets. For the remaining objects, we computed stellar mass estimates on the basis of log g and stellar radius, when available, or from the exoplanet's orbital period and semimajor axis, when available. This process yielded a total of 834 classifiable objects.

The mass of each exoplanet was obtained from one of three archive fields: planet mass, $M \sin i$, or planet radius, in that order. We eliminated five exoplanets for which none of these fields were listed, leaving 829 entries. When only planet radius was listed, we used a radius–mass relationship to estimate the mass. The results are robust against the choice of the radius–mass relationship. The semimajor axis of each exoplanet was either available from the archive or computed on the basis of orbital period and host star mass. We used the resulting values in Equation (9) to test whether non-Kepler exoplanets have a mass that exceeds the corresponding orbit-clearing mass (Figure 4).

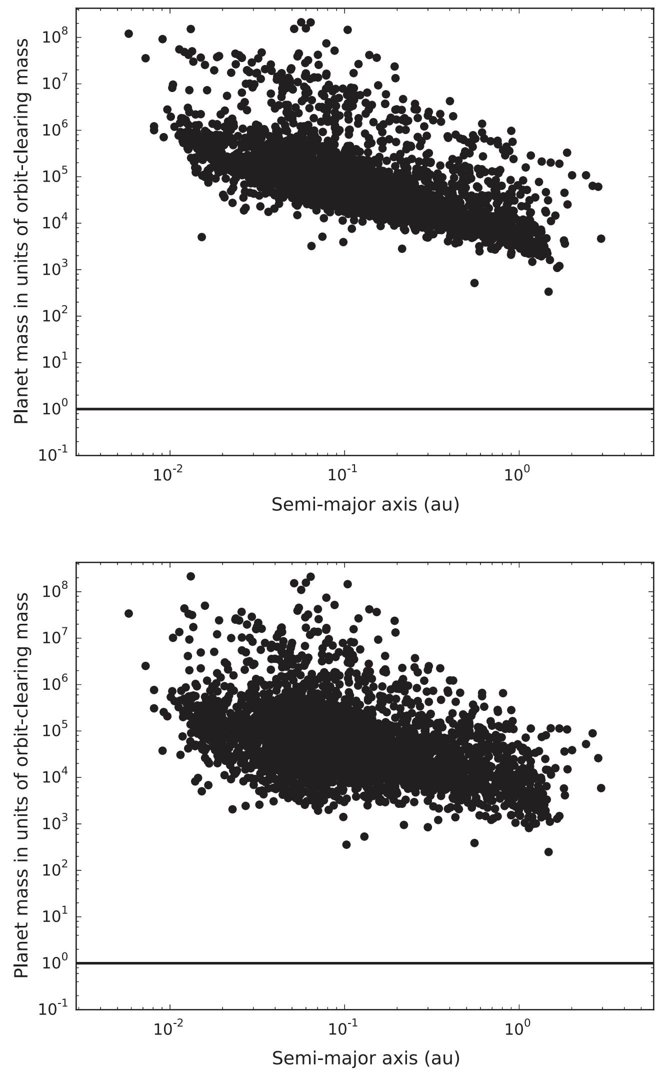

Figure 3. Planet test ($C = 2\sqrt{3}$, $t_\star = t_{MS}$) applied to 4276 Kepler objects categorized as either "confirmed" or "candidate" in the NASA Exoplanet Archive. All objects have a mass that exceeds the mass required to clear the corresponding orbital zone over the lifetime of the host star on the main sequence ($\Pi \geqslant 1$). Results are robust against the choice of the radius–mass relationship. Two such relationships are shown: (top) Fang & Margot (2012) and (bottom) Fabrycky et al. (2014).

4.3. Pulsar Planets

The choice of the characteristic time scale $t_\star$ for neutron stars is delicate. The lifetime of their progenitors on the main sequence is not directly accessible. The characteristic age of the pulsar, while easily measurable from the period and period derivative, can be very short (~10^4 years) and not representative of the time over which planets can clear their orbits. For ease of implementation, one could adopt $t_\star = 4.3 \times 10^9$ years corresponding to the main sequence lifetime for a star of mass 1.4 $M_\odot$, equivalent to the Chandrasekhar limit. In applying our test to pulsar planets, we used time scales of 10^9–10^{10} years. As of 2015 July 17, the NASA Exoplanet Archive included five pulsar planets. We assumed neutron star masses of 1.4 $M_\odot$ and used Equation (8) to test whether all five objects have a mass that exceeds the corresponding orbit-clearing mass (Figure 5).

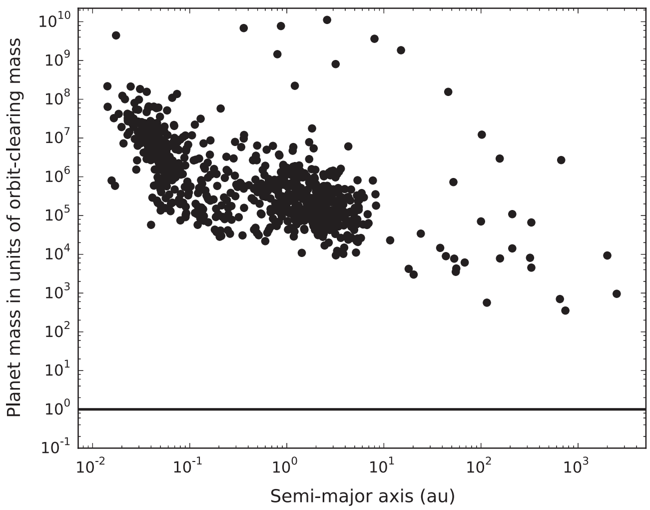

Figure 4. Planet test ($C = 2\sqrt{3}$, $t_\star = t_{\rm MS}$) applied to 829 confirmed, non-Kepler exoplanets in the NASA Exoplanet Archive. All objects have a mass that exceeds the orbit-clearing mass ($\Pi \geqslant 1$).

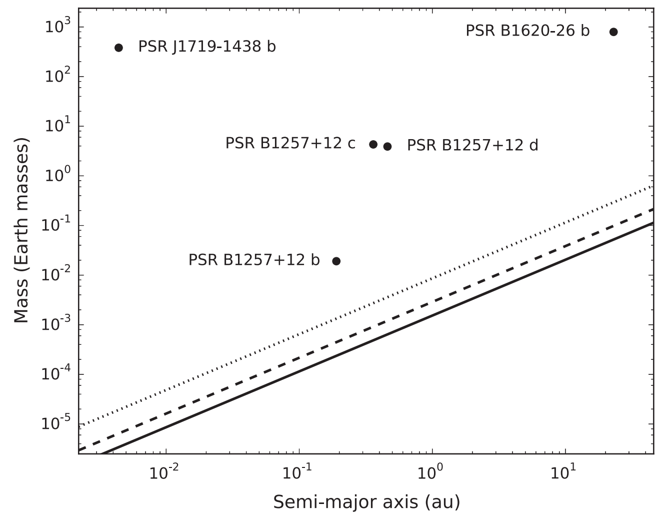

Figure 5. Masses of pulsar planets compared to the mass required to clear the corresponding orbital zone for a host star of mass 1.4 $M_\odot$. Lines show clearing to $2\sqrt{3}$ Hill radii in 10 billion years (solid line), 4.3 billion years (dashed line), or 1 billion years (dotted line).

5. MAXIMUM MASS

The 2006 IAU planet definition does not specify an upper limit for the mass of a planet. However, the IAU Working Group on Extrasolar Planets (WGESP), part of the former Division III Planetary Systems Science, wrote a position statement addressing the need to differentiate planets from brown dwarfs (Boss et al. 2007). In its working definition, the WGESP used the limiting mass for thermonuclear fusion of deuterium, nominally 13 Jupiter masses, to demarcate objects classified as planets from objects classified as brown dwarfs. Free-floating objects are also considered in the WGESP working definition and are never classified as planets.

The WGESP's working definition is based on an observable physical quantity, which makes classification straightforward. The mass of the object to be classified is compared to the limiting mass for deuterium fusion. This criterion does not require that the object actually experienced deuterium fusion. This distinction is critical because it is far more practical to measure the mass of an object than to understand its evolutionary history. In doing so, the WGESP acknowledged that it was preferable to risk a small fraction of inaccurate classifications (e.g., classifying an object that experienced deuterium fusion at some point in its history as a planet) than to build a classification around the detection of a signature that is not reliably observable. The criterion proposed in this paper is very much aligned with the WGESP's philosophy. The mass of the object to be classified is compared to the corresponding orbit-clearing mass via the planet discriminant Π. The criterion does not require that the object actually cleared its orbit.

6. ASTROPHYSICS OF PLANET FORMATION

In accordance with the IAU's and WGESP's approaches, we purposefully developed a taxonomic tool that is not based on hypotheses related to planet formation. We chose not to allow our incomplete and possibly incorrect understanding of the planet formation process to interfere with the design of a planet discriminant. Concordance of the classification scheme with formation and evolution processes may ultimately become a desirable trait, but it is only one of several desirable traits of a good taxonomy.

In spite of our agnosticism, the planet discriminant appears to identify actual physics of the planet formation process. For instance, there is a gap of more than three orders of magnitude in the value of Π between planets and dwarf planets in the solar system. This gap may illuminate the physics of accretion time scales, oligarchic growth, and dynamical evolution of the solar system (e.g., Armitage 2013). Likewise, all classifiable exoplanets appear to have Π values well above 1. Although this characteristic may currently be due to observational selection effects, persistence of this trait in future surveys with more sensitive instruments would compel us to seek an explanation.

7. DISCUSSION

7.1. Extent of Orbital Zone to be Cleared

We adopted the size of a planet's feeding zone, corresponding to $C = 2\sqrt{3}$, as the minimum extent of the orbital zone to be cleared. This theoretical value is in agreement with the results of numerical simulations. Morrison & Malhotra (2015) described the boundaries of orbital zones that are cleared over a wide range of planet-to-star mass ratios (10^{-9}–$10^{-1.5}$) and planet radii (0.001 $R_{\rm H}$–0.1 $R_{\rm H}$) in the context of the planar, circular, restricted three-body problem. The boundaries remain within a factor of 0.6–1.5 of $2\sqrt{3}R_{\rm H}$ over this entire range of conditions. Similar extents may apply to orbits with eccentricities up to about 0.3 (Quillen & Faber 2006). In non-planar systems of multiple interacting planets, the extent of the zone that is cleared may of course exceed $2\sqrt{3}R_{\rm H}$ because of additional perturbations.

We also evaluated the planet discriminant for a slightly larger extent, corresponding to $C = 5$, which roughly matches half the observed separations between tightly packed exoplanets. Most planets in Kepler multi-planet systems are separated by at least 10 mutual Hill radii[2] with a mean spacing of $\sim$20 mutual Hill radii (Fang & Margot 2013; Lissauer et al. 2014). The value of 10 mutual Hill radii also represents the

[2] The mutual Hill radius for two planets has the same form as Equation (3) but involves the sum of the planetary masses and the average of the semimajor axes.

approximate minimum spacing required for long-term dynamical stability of planets on circular, coplanar orbits in multi-planet systems (Chambers et al. 1996; Pu & Wu 2015, and references therein).

It is noteworthy that the orbit-clearing criterion does not depend strongly on the adopted value of C; other values besides $2\sqrt{3}$ and 5 may be considered.

7.2. Characteristic Time Scale

We adopted the lifetime of the host star on the main sequence as a sensible time scale, but other choices can be readily accommodated. Restricting the time scale to 10% of the stellar lifetime, for instance, would increase the orbit-clearing mass by a factor of $\sim$6. Doing so would not change the classification of any of the objects considered here.

7.3. Clearing Process

Numerical simulations in the context of the planar, circular, restricted three-body problem can reveal the fate of particles that orbit in the vicinity of a planet (Morrison & Malhotra 2015). For small values of the planet-to-star mass ratio, simulations show that removal by collisions with the planet is more frequent than removal by scattering, although scattering remains dominant in the limit of small planetary radii. In addition, removal times are shorter when collisions are the dominant clearing mechanism (Morrison & Malhotra 2015). An object with $\Pi \geqslant 1$ can therefore clear its orbital zone within the prescribed time scale, whether collisions or scattering events prevail in the clearing process. We prefer the simplicity of a single lower bound provided by the diffusive time scale over the construction of a hybrid criterion with both collisional and diffusive time scales (e.g., Levison 2008).

7.4. Bodies on Eccentric Orbits

Although the orbit-clearing criterion was developed for planets on circular orbits, the essence of the metric is based on a random walk in orbital energy. Gravity kicks will modify the orbits of small bodies regardless of the planet's eccentricity, such that the basic concept of orbit clearing remains applicable. How the orbit-clearing time scale varies as a function of orbital eccentricity is an interesting question left for future work. Because we anticipate that the criterion will hold to first order and because we favor ease of implementation, we suggest applying the planet discriminant Π regardless of orbital eccentricity.

7.5. Co-orbitals

A planet can clear its orbit yet capture small bodies in tadpole, horseshoe, and quasi-satellite orbits near the Lagrange equilibrium points (Murray & Dermott 1999). Well-known examples include the Trojan asteroids associated with Jupiter. Our proposed criterion follows the IAU definition in that the existence of bodies in such orbits has no bearing on the classification. More exotic configurations can be handled by the proposed criterion as well. Co-orbital planets have not been discovered to date, but some configurations are in principle stable over long periods of time (Salo & Yoder 1988; Smith & Lissauer 2010). For instance, one could differentiate between co-orbital planets (where the planet masses individually exceed M_{clear}) and a planet-trojan system (where only one of the bodies meets the criterion).

7.6. Satellites

The IAU has not formally defined satellites, which are informally understood to be celestial bodies that orbit planets, dwarf planets, or asteroids. Satellites to planets will have little or no impact on the classification if the satellite-to-planet mass ratio is low. At higher values of the mass ratio, satellites may affect the classification because it is the sum of the component masses in a bound system that determines the ability to clear an orbital zone. The terminology could in principle differentiate between two-body planets (where the sum of the masses exceeds M_{clear}, but the individual component masses do not) and double planets (where the individual masses both exceed M_{clear}).

Improvements to the classification are needed to deal with celestial bodies that orbit brown dwarfs. Because such bodies do not orbit a star or stellar remnant, we do not consider them planets.

7.7. Circumbinary Planets

Celestial bodies in orbit around a system of bound stars can be classified with the proposed criterion by using the sum of the stellar masses in Equation (8) and a time scale corresponding to the shortest stellar lifetime.

7.8. Free-floating Objects

In conformity with the 2006 IAU planet definition and WGESP recommendations, we do not consider planetary objects that never orbited a star or no longer orbit a star (i.e., free-floating objects) to be planets. Some scientists dislike the concept of a planet definition that depends on context and would prefer to focus on intrinsic properties. However, there are instances in which context justifiably prevails in the classification (e.g., asteroid versus meteorite, magma versus lava, cloud versus fog), and there is no reason to dispense with a useful distinction in the taxonomy of planets.

7.9. Migration and Scattering

An object that is classified as a planet, with $\Pi \geqslant 1$, will lose this classification if it migrates or is scattered to a distance from its host star where $\Pi < 1$. This reclassification is similar to the reclassification of asteroids as meteorites upon impact with a planet or moon. According to the proposed criterion, an Earth-mass body orbiting a solar-mass star at 400 au, where $\Pi < 1$, would not be considered a planet, whether it formed there or was transported there after formation.

7.10. Advantage Over Other Proposed Metrics

Soter (2006) proposed a planet discriminant that requires the mass of a body to be 100 times the mass of all other bodies that share its orbital zone. Levison (2008) favored a definition in which an object that is part of a smooth size distribution is not a planet. Valsecchi (2009) proposed a criterion in which the mass of a body must exceed the mass of all bodies that come close to or cross its path by a factor of 1000. The difficulty with implementing these criteria is that the mass or size distribution of the neighboring small bodies must be measured or estimated. In other words, it is not possible to classify a celestial body until knowledge about neighboring bodies is secured. It would, therefore, be difficult to classify most exoplanets based on these definitions. The main advantage of the orbit-clearing criterion proposed here is that no such knowledge is required.

8. ON ROUNDNESS

Equation (9) provides a quantifiable criterion that addresses the first and third aspects of the 2006 IAU planet definition. A separate issue is whether the second requirement, i.e., roundness, is necessary. It is possible that every object that can clear its orbital zone is "nearly round," which would make the roundness requirement superfluous.

Tancredi & Favre (2008) examined the size and density bounds that guarantee approximate roundness in planetary bodies. They recommended a diameter threshold of 800 km for rocky bodies, which corresponds to a mass threshold of $M_{\rm round} \sim 10^{-4} M_\oplus$ for a density of 2.5 g cm^{-3}. Lineweaver & Norman (2010) find a slightly smaller diameter threshold of 600 km.

A planet that has cleared its orbital zone is expected to have accumulated a mass on the order of the isolation mass, the mass of the planetesimals in its feeding zone. The mass of the planet may be substantially larger than the isolation mass if there has been migration and resupply of disk material, migration of the planet through the disk, planet mergers, post-formation accretion of asteroidal material, or a combination of these factors.

An expression for the isolation mass adapted from Armitage (2013) reads

$$\frac{M_{\rm iso}}{M_\oplus} = 6.6 \times 10^{-2} \left(\frac{M_\star}{M_\odot}\right)^{-1/2} \left(\frac{\Sigma_{\rm p}}{10 \text{ g cm}^{-2}}\right)^{3/2} \left(\frac{a_{\rm p}}{1 \text{ au}}\right)^{3}, \tag{11}$$

where $\Sigma_{\rm p}$ is the local surface density of the planetesimals. The functional form of the surface density is uncertain, but a common model is

$$\Sigma_{\rm p} = \Sigma_0 \left(\frac{a_{\rm p}}{1 \text{ au}}\right)^{-3/2}, \tag{12}$$

which yields an isolation mass

$$\frac{M_{\rm iso}}{M_\oplus} = 6.6 \times 10^{-2} \left(\frac{M_\star}{M_\odot}\right)^{-1/2} \left(\frac{\Sigma_0}{10 \text{ g cm}^{-2}}\right)^{3/2} \left(\frac{a_{\rm p}}{1 \text{ au}}\right)^{3/4}. \tag{13}$$

Evaluation of this expression over a wide range of conditions ($1 < \Sigma_0 < 10$ g cm^{-2}, $0.01 < a_{\rm p} < 100$ au) shows that

$$M_{\rm iso} > M_{\rm round}. \tag{14}$$

This approximate calculation suggests that every object that has cleared its orbital zone and accumulated at least an isolation mass worth of material is nearly round. Because of residual uncertainties related to the size at which planetary bodies become round, the exact surface density profile of planetesimals, and the process of planet formation, it is difficult to gauge roundness on theoretical grounds with greater certainty. However, attempting to gauge roundness observationally would be equally difficult and lead to comparable uncertainties. The threshold for roundness depends on the interior composition of the body and temperature-dependent material strength (Tancredi & Favre 2008), which are not observable from Earth.

9. POSSIBLE IMPROVEMENTS TO THE IAU PLANET DEFINITION

Because a quantitative orbit-clearing criterion can be applied to all planets and exoplanets, it is possible to extend the 2006 IAU planet definition to stars other than the Sun and to remove any possible ambiguity about what it means to clear an orbital zone.

In addition, because it is probable that all objects that satisfy the orbit-clearing criterion also satisfy the roundness criterion, it is possible to simplify the definition by removing the latter criterion.

One possible formulation (with $C = 2\sqrt{3}$, $t_\star = t_{\rm MS}$) is as follows.

> A planet is a celestial body that (a) is in orbit around one or more stars or stellar remnants, (b) has sufficient mass to clear (or dynamically dominate) the neighborhood around its orbit, i.e., $\Pi \geqslant 1$, (c) has a mass below 13 Jupiter masses, a nominal value close to the limiting mass for thermonuclear fusion of deuterium.
>
> For single-star systems, $\Pi \geqslant 1$ when
>
> $$\frac{M_{\rm p}}{M_\oplus} \gtrsim 1.2 \times 10^{-3} \left(\frac{M_\star}{M_\odot}\right)^{5/2} \left(\frac{a_{\rm p}}{1 \text{ au}}\right)^{9/8},$$
>
> where M is mass, a is semimajor axis, and subscripts p, $\star$, $\oplus$, $\odot$ refer to the planet, star, Earth, and Sun, respectively.

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